A cooperative guidance law for target estimation by multiple unmanned aerial vehicles

W Lee¹*, H Bang¹, and H Leeghim²

¹Division of Aerospace Engineering, Korea Advanced Institute of Science and Technology, Daejeon, Republic of Korea
²Samsung Electronics, Suwon, Republic of Korea

The manuscript was received on 21 October 2010 and was accepted after revision for publication on 25 February 2011.

DOI: 10.1177/0954410011404098

Abstract: A new cooperative guidance law is proposed for target estimation of multiple unmanned aerial vehicles (UAVs) without involving numerical computational work. The guidance law derivation is based upon the Fisher Information Matrix (FIM) to quantify the amount of target information. For two UAVs, an analytical one-step determinant of FIM is introduced, which sufficiently reflects the trend of previous well-known optimal manoeuvres. Therefore, by considering only one-step information, as opposed to accumulated total information, a new feedback guidance law with intuitive physical analysis is induced and then its validity is verified by comparing it with the results of previous approaches. To apply the proposed idea to more multiple UAVs, a generalized formulation is derived as a weighted combination of each guidance law. Also, for more realistic application, the proposed guidance law is modified to deal with an avoidance problem of risk zone.

Keywords: feedback cooperative guidance law, cooperative estimation, one-step determinant of Fisher Information Matrix

I. INTRODUCTION

Target estimation problems using the measurements of passive bearing sensors such as vision sensors and infrared sensors have been studied extensively for several decades in robotics, guided weapons, and unmanned aerial vehicles (UAVs) areas. This subject is associated with useful applications in tracking or localizing for different missions including exploration, surveillance, and reconnaissance. Owing to noisy sensor measurements, even if the target states can estimated without UAVs manoeuvres, exploiting desired UAVs motions may be worthwhile to enhance estimation performance [1, 2]. Therefore, the generation of appropriate observer trajectories for such problems has received considerable attention in recent studies.

There have been some efforts to solve the target localization problem using the optimal control theory to design observer trajectories. The Fisher Information Matrix (FIM) was applied to express the amount of information as a criterion in the optimization problem formulation. More information implies that error covariance becomes smaller as explained by the Cramer–Rao lower bound (CRLB). Thus, the inverse of FIM is essentially equivalent to the lower bound of the error covariance [3, 4]. Using the FIM measure, the sequence of observer courses to maximize target information can be constructed. To design such optimal observer courses, Oshman and Davidson [1] proposed an optimal control approach based on parameter optimization using a fixed set of constant parameters. Passerieux generated optimal trajectories in terms of a finite number of legs [5, 6] by applying the Hamiltonian–Jacobi equation of optimal control theory with the FIM dynamics [2]. Liu [7]
derived an optimal course by using the Hamiltonian–Jacobian equation also. Moreover, optimal paths for a target localization using UAVs to reduce radar cross-section or to maximize the probability of target attack have been investigated [8, 9].

Unlike such optimization approaches by off-line process, other studies have been focused on real-time path planning methods. The receding horizon control (RHC) approach for finite time-step optimization was explored to generate optimal trajectories with less computational load [10–13]. In reference [14], an iterative step method was proposed to minimize local cost for real-time operation. In reference [15], simultaneous estimation and optimization approach using a gradient descent method was proposed for the three-dimensional (3D) bearings-only target localization problem for real-time operations. In addition, optimal sensor placement of ground moving target indicator by a gradient-based search technique was studied in reference [16].

Recently, as an information-theoretic approach for sensor placement has been under increasing spotlight, real-time control considering other information measures as well as FIM are investigated. In reference [17], mutual information between sensors and target states using the particle filter was developed, and the control input was obtained by an iterative algorithm using the mutual information. In reference [18], entropy of a probability distribution was employed instead of the mutual information for computational ease. In addition, based on the information-theoretic approach, there are other interesting approaches to derive guidance laws for alleviating computational burden. Choi and How [19] derived a steering law using the gradient of a smoother form of mutual information rate for continuous paths. In reference [20], a gradient ascent law was introduced based on the FIM using the identical gradient approach. Also, Grocholsky [21] developed an algebraic form of optimal solution to maximize expected entropy information for range-only localization. However, it is not easy to understand rather intuitive physical analysis of the above approaches due to complex expressions. On the other hand, Sinclair and Prazenica [22] derived an optimal guidance law only for two UAVs using a geometric method in the form of previously well-known optimal maneuvers as an orthogonal maneuver and a forward maneuver. However, even though this approach may be easy for physical interpretation, it is necessary to reformulate the guidance law for more UAVs case.

Therefore, in this article, a new and less complex cooperative guidance law in a feedback form for real-time path planning without reformulation for N UAVs case is proposed based on the information-theoretic approach. The key idea is to apply an analytical equation of one-step determinant of FIM to an indirect optimization problem. This could be considered useful since the one-step determinant reflects sufficiently the trend of optimal cooperative maneuvers for target estimation. Therefore, the cooperative guidance law is expected to generate an explicit command which maximizes the next-step information of a target at every time although the obtained results cannot be claimed to be global optimal from the perspective of the total trajectory. To prove this expectation, its results are compared with previous results derived by considering accumulated information using RHC approach. The command from the proposed guidance law is derived as a weighted combination between orthogonal and forward maneuver, which is analogous to the results of references [10] and [22]. However, while the guidance law using the geometrical method in reference [22] should be artificially tuned for a weighting parameter between the orthogonal maneuver and the forward maneuver to match the optimal trajectories, the proposed guidance law in this study can tune the weighting parameter automatically to maximize the next-step information by optimization. Also, since the proposed cooperative guidance law in this article is generalized for more multiple vehicles, the guidance law does not have to be reformulated depending on the number of UAVs. Therefore, even if the number of UAVs is varied during maneuvers, the proposed cooperative guidance law is able to generate explicit command adaptively accommodating the change in the number of UAVs. Additionally, because this guidance law generates the command to maximize target information of next-step at every time, it is applicable to even an arbitrarily moving target following the trend of optimal maneuvers.

This article is organized as follows: in the first section, a general localization problem is introduced and the characteristics of error covariance ellipsoid and the FIM are analysed for a cooperative maneuver by two UAVs. Following this, the so-called one-step determinant of the FIM is formulated for optimization, which shows similar trend to the optimal property represented by orthogonal and forward maneuvers. In the next section, the one-step determinant of FIM is applied to the optimization problem formulation to maximize the information by two UAVs during the one-step time period. Using the Hamiltonian–Jacobi equation, the new cooperative guidance law is derived explicitly in a feedback form only for the one-step time frame of maneuvers. To extend the guidance law to multiple number of UAVs, a sequence rule for the determinant of the FIM for several UAVs is established and then a general.
formulation of the guidance law for \( N \) UAVs is derived. For more practical application, the guidance law is modified to handle the risk-zone avoidance problem. In the final section, various simulations are conducted to verify the proposed cooperative guidance law. Also, in order to prove the validity of the guidance law, its results are compared with those of previous approaches, which consider total accumulated information by RHC method. Finally, sensitivity of the proposed guidance law to the estimated target states instead of true states in real operation is analysed.

2 PROBLEM DEFINITION

A general localization problem using multiple UAVs with only a bearing sensor is presented in Fig. 1. UAVs measure the bearing angle, \( \beta \), of the line of sight to a target. From the history of the bearing angle, UAVs can estimate the target coordinates using a cooperative network. It is assumed that the target remains on the ground level, and the UAVs move at a constant speed, denoted here as \( v \), with identical altitude, (same horizontal plane), so that the UAVs are able to estimate the target information from bearing measurements with the height information already known. In this study, to focus on the introduction of a new cooperative guidance law for multiple UAVs, some realistic constraints are not considered, such as limitation in the field of view of the sensors and range restriction of cooperative network between the UAVs. Additionally, each UAV is supposed to share states of each other and a target information by cooperative network every time. Kinematics equations of UAVs can be simply expressed as follows

\[
\dot{X}(t) = \begin{bmatrix} \dot{x}_j(t) \\ \dot{y}_j(t) \end{bmatrix} = \begin{bmatrix} v \cos u_j(t) \\ v \sin u_j(t) \end{bmatrix} = f, \quad j \in [1, 2] \quad (1)
\]

where \( j \) represents the number of UAVs, and \( u_j \) stands for the heading input determined by the cooperative guidance law in the next section. The measurement variable, \( \beta \), equation is

\[
\beta(t) = \arctan \left( \frac{y_j(t) - y(t)}{x_j(t) - x(t)} \right) + \nu(t)
\]

where the subscript \( j \) denotes the target states, \( \nu(t) \) is zero-mean and uncorrelated Gaussian white noise with a constant variance \( \sigma^2 \). To achieve the desired estimation of the target location, UAVs should manoeuvre to minimize the target estimation error covariance. Alternatively, it can be explained as maximization of the information of target. Also, this can be interpreted by the CRLB theory, in which the inverse of the FIM is equivalent to the lower bound of the estimation error covariance. FIM is also used as a criterion for the amount of information

\[
P = E[(\hat{\beta}(t) - \beta(t))(\hat{\beta}(t) - \beta(t))^T] \geq F^{-1}
\]

where \( \beta(t) \) denotes a non-random parameter, \( \hat{\beta} \) an unbiased estimator, and \( \beta \) a set of measurements. FIM, \( F \), is derived as the partial derivative of the logarithm of a likelihood function \( [23] \) as follows

\[
F = E\left[ \frac{\partial}{\partial \beta} \ln p(\beta|\mathbf{x}) \right]_{\beta=\beta_T}
\]

where \( p(\beta|\mathbf{x}) \) is a conditional probability density function when \( \mathbf{x} \) becomes the target states \( \mathbf{x}_T \). If a target is stationary, its trajectory is characterized by a two-dimensional (2D) position state vector such that

\[
\mathbf{x}_T(t) = \begin{bmatrix} x_T(t) \\ y_T(t) \end{bmatrix}
\]

Then, in this bearing only localization problem, the FIM can be modified as follows \([1]\)

\[
F_{ij}(t) = \begin{bmatrix} \sum_{t \in [t]} \frac{\Delta y_j(t)}{\sigma_j^2(t)} r_j^2(t) \Delta y_j(t) \\ -\sum_{t \in [t]} \frac{\Delta y_j(t)}{\sigma_j^2(t)} r_j^2(t) \Delta y_j(t) \\ \sum_{t \in [t]} \frac{\Delta y_j(t)}{\sigma_j^2(t)} r_j^2(t) \Delta y_j(t) \\ -\sum_{t \in [t]} \frac{\Delta y_j(t)}{\sigma_j^2(t)} r_j^2(t) \Delta y_j(t) \end{bmatrix}
\]

where the subscript \( j \) stands for the identification of each UAVs and

\[
\Delta x_j(t) = x_T(t) - x_j(t) \\
\Delta y_j(t) = y_T(t) - y_j(t) \\
r_j^2(t) = \Delta x_j^2(t) + \Delta y_j^2(t)
\]

Also, it is well known for that the \( N \) vehicles, the total information from all vehicles can be expressed as a summation of the individual information as follows

\[
F(t) = \sum_{j=1}^{N} F_{ij}(t)
\]
In order to represent the effect of cooperative maneuvers, an error covariance ellipsoid is introduced near the target as shown in Fig. 2. If each UAV shares the information of the target, then the uncertainty of the target becomes smaller by the common uncertainty region displayed in dark colour. Moreover, it is apparent that the magnitude of the uncertainty ellipsoid is inversely proportional to the level of orthogonality of both UAVs as shown in Figs 2(a) and (b) due to the inherent characteristics of bearing sensors. Therefore, under cooperative manoeuvres with two UAVs, behaviours maintaining orthogonal relationship are the optimal manoeuvre to maximize the information of the target. Additionally, since each error covariance area becomes smaller as the distance between a sensor and its target decreases, if two UAVs moving to the target maintain the orthogonal manoeuver, the total information of the target can be certainly maximized during the full manoeuvre period.

The above analysis can be validated mathematically by considering the total FIM for only one-step period. Under cooperative manoeuvres by two UAVs, the total FIM for only the one-step period can be formulated as

\[
F(t) = \begin{bmatrix}
\frac{\sin^2 \beta_1}{\sigma_1^2(t)r_1^2(t)} & \frac{\sin^2 \beta_2}{\sigma_2^2(t)r_2^2(t)} & \frac{\sin \beta_1 \cos \beta_1}{\sigma_1^2(t)r_1^2(t)} & \frac{\sin \beta_2 \cos \beta_2}{\sigma_2^2(t)r_2^2(t)} \\
\frac{\sin \beta_1 \cos \beta_1}{\sigma_1^2(t)r_1^2(t)} & \frac{\sin \beta_2 \cos \beta_2}{\sigma_2^2(t)r_2^2(t)} - \frac{\cos^2 \beta_1}{\sigma_1^2(t)r_1^2(t)} & \frac{\cos^2 \beta_2}{\sigma_2^2(t)r_2^2(t)}
\end{bmatrix}
\]

In this study, to quantify the information level, the determinant of the FIM \( \det(F(t)) \) is taken as a scalar measure [1, 10]. Since \( \det(F(t)) \) is identical to the multiplication of its eigenvalues, it becomes an equivalent scalar amount of the inverse of uncertainty ellipsoid area [15]. Therefore, this criterion could be considered as an appropriate measure of the quantification of information level. By the definition of \( \det(F(t)) \) during the one-step time period, a simple equation is derived analytically as follows

\[
\det F(t) = \frac{\sin^2 (\beta_1(t) - \beta_2(t))}{(\sigma_1 \sigma_2 r_1(t)r_2(t))^2}
\]

In order to maximize \( \det(F(t)) \), both UAVs should manoeuvre to reduce the distance between each UAV and the target, and to make the angle between the two vectors (from each UAV to the target) orthogonal. This is because the accuracy of the target information by the bearing sensor would improve if the sensor approaches closer to the target and if both sensors measure the target more orthogonally to each other. Note that in this study, the covariance characteristics of each sensor is set to be identical in order to investigate only the effect of the distance and the bearing angle. Through a trade-off between the distance and the bearing angle; therefore, the amount of information can be maximized.
of cooperative path planning [10, 22]. Since one can see that this one-step \text{det}(FIM) reflects the characteristics of optimal cooperative manoeuvres of two UAVs, even though only the one-step \text{det}(FIM) is considered instead of total accumulated FIM, it is expected that effective trajectories with analogous behaviour to the optimal results can be constructed. Based on this analysis, hence, a new cooperative guidance law is derived in the next section.

3 COOPERATIVE GUIDANCE LAW DESIGN

As mentioned before, because the analytical one-step \text{det}(FIM) reflects the behaviour of the optimal cooperative manoeuvres, a new guidance law based on this fact is induced through an indirect optimization approach. Although the generated command history from the approach is not globally optimal in terms of the total trajectory, since the trend of the optimal trajectory could be reflected sufficiently, effective and proper commands can still be expected. To evaluate \text{det}(FIM) during the one-step period with kinematic constraint of UAVs, a performance index is expressed as follows

\[ J = -\text{det} \mathbf{F}(t + \Delta t) + \int_{t}^{t+\Delta t} \lambda^T(f - \dot{X})d\tau \tag{11} \]

In the above performance index, \(-\text{det} \mathbf{F}(t + \Delta t)\) is regarded as a soft constraint, \(\phi\), in general optimal control problems. The Hamiltonian is expressed as [24]

\[ H = \lambda^T \mathbf{f} = \left[ \lambda_{xj} v \cos u_j + \lambda_{yj} v \sin u_j \right], \quad j \in [1, 2] \tag{12} \]

Additionally, optimality condition can be induced from Hamiltonian such that

\[ H_u = \lambda^T f_u = \begin{bmatrix} -\lambda_{xj} v \sin u_j + \lambda_{yj} v \cos u_j \\ -\lambda_{yj} v \sin u_j + \lambda_{yj} v \cos u_j \end{bmatrix} = \mathbf{0} \tag{13} \]

From the optimality condition, the guidance law is derived as

\[ u_j = \tan(\alpha_{jk}), \quad j \in [1, 2] \tag{14} \]

In order to satisfy \(H_{uu} > 0\) for a minimization problem, the costate, \(\lambda\) sign should be negative in this case. Also, all the costates are constant to be consistent with the costate dynamics

\[ \dot{\lambda} = -H_u^T = \mathbf{0} \tag{15} \]

Hence, the final costates, \(\lambda_f\) should satisfy

\[ \lambda_f^T = \lambda^T = \phi_x \tag{16} \]

After some algebra, each final costate can be analytically expressed simply as

\[ \lambda_{xj} = -\frac{2\sin \Delta \beta_{jk}}{(\sigma \beta_{jk})^2} \left[ \cos \Delta \beta_{jk} \cdot \Delta y_j + \sin \Delta \beta_{jk} \cdot \Delta x_j \right] \]

\[ \lambda_{yj} = -\frac{2\sin \Delta \beta_{jk}}{(\sigma \beta_{jk})^2} \left[ -\cos \Delta \beta_{jk} \cdot \Delta x_j + \sin \Delta \beta_{jk} \cdot \Delta y_j \right] \]

where

\[ \Delta \beta_{jk} = \beta_j - \beta_k, \quad j, k \in [1, 2], \quad j \neq k \tag{17} \]

With the costates substituted into equation (14), the final form of the guidance law is rewritten in an analytical form

\[ u_j = \tan(\alpha_{jk}), \quad j \in [1, 2], \quad j \neq k \tag{18} \]

and \(\alpha_{jk} = -\text{sgn}(\sin \Delta \beta_{jk}), \quad j, k \in [1, 2], \quad j \neq k \tag{19} \)

and ‘\text{sgn}’ represents a signum function. Note that even if the coefficients of each costate are cancelled out, the sign should be preserved because it determines the direction of the guidance law. From this guidance law, explicit command can be generated to maximize the target information at the next step based upon the current situation. In equation (19), the guidance law is constructed easily using the current measurements of both UAVs and the relative distance between estimated target position and each UAV location in a feedback form. Therefore, the guidance law offers advantages in practical implementation with negligible computational load and inherent robustness to external disturbances due to the nature of feedback control.

3.1 Interpretation of the cooperative guidance law

For validation of the cooperative guidance law equation (19), its physical significance should go through close analysis. First, the trigonometric functions in front of \(\Delta x_j\) and \(\Delta y_j\) play the role of weighting parameters based on the geometric relationship. Hence, if both UAVs are on the same line to the target, \(\Delta \beta_{jk}\) reduces to zero, and the guidance law becomes

\[ u_1 = \tan(\Delta x_1, \Delta y_1), \quad u_2 = \tan(\Delta x_2, -\Delta y_2) \tag{21} \]
Physical meaning of the cooperative guidance law

This is apparently an orthogonal manoeuvre command because both UAVs are not orthogonal to each other. In Fig. 3, line (A) represents the manoeuvre command direction. Such an orthogonal manoeuvre is performed to compensate for the limitation of bearing sensors, for which information in range direction is relatively poor compared to cross-range direction. On the other hand, if each UAV is orthogonal with $\Delta \beta_{jk} = \pi/2$, the guidance law becomes

$$u_l = \arctan2(\Delta y_j, \Delta x_j), \quad j \in [1, 2] \quad (22)$$

Because both UAVs are already orthogonal to each other, only the forward manoeuvre command to reduce the distance between each UAV and the target is certain issued as described by line (B) in Fig. 3. This is because the estimation efficiency of the bearing sensor is improved as the distance between the UAV and target becomes shorter. Unlike the special cases above, a general case is considered here where $\Delta \beta_{jk} = \pi/4$. As the bearing angle relationship is in the middle between the above special cases, the orthogonal manoeuvre command and the forward flight command are generated at one-to-one ratio

$$u_l = \arctan2(-\Delta x_j + \Delta y_j, \Delta y_j + \Delta x_j),$$
$$u_2 = \arctan2(\Delta x_j + \Delta y_j, -\Delta y_j + \Delta x_j) \quad (23)$$

Consequently, the cooperative guidance law is expected to comprise both orthogonal and forward manoeuvres with a weighting parameter determined by the bearing angle. This exactly agrees with the previous analysis in equation (10). Therefore, this new guidance law is valid to reflect the trend of optimal cooperative manoeuvres which are obtained by a trade-off between orthogonal manoeuvre and forward manoeuvre. Since this intuitive physical analysis is helpful to anticipate how a command is generated, the proposed guidance law is easier and useful in the practical implementation than the previous methods with complicated expressions, as introduced in references [19] and [21]. In addition, it is worthwhile to note that while a gain tuning method was applied in order to account for the orthogonal manoeuvres in the previous study [22], the weighting parameters for the orthogonal manoeuvre are analytically determined by optimization in our approach. Consequently, in real-time, the weighting parameters to maximize the information of the target at the next step can be tuned adaptively in every situation.

3.2 General formulation of cooperative guidance law for multiple UAVs

Based on the previous guidance law for two UAVs, it can be further extended to a general formulation of multiple UAVs. To develop the general formulation, an inductive approach is taken to derive a generalized one-step det(FIM) before handling the guidance law. In order to find a sequence rule, an one-step det(FIM) for three UAVs is expressed

$$\text{det} \mathbf{F} = \frac{\sin^2 \beta_1 + \sin^2 \beta_2 + \sin^2 \beta_3}{\sigma_1^2 r_1^2 + \sigma_2^2 r_2^2 + \sigma_3^2 r_3^2}$$
$$- \left( \frac{\sin \beta_1 \cos \beta_1 + \sin \beta_2 \cos \beta_2 + \sin \beta_3 \cos \beta_3}{\sigma_1^2 r_1^2} + \frac{\sin \beta_1 \cos \beta_1 + \sin \beta_2 \cos \beta_2 + \sin \beta_3 \cos \beta_3}{\sigma_2^2 r_2^2} + \frac{\sin \beta_1 \cos \beta_1 + \sin \beta_2 \cos \beta_2 + \sin \beta_3 \cos \beta_3}{\sigma_3^2 r_3^2} \right)^2$$
$$= \sin^2(\beta_1 - \beta_2) + \sin^2(\beta_2 - \beta_3) + \sin^2(\beta_3 - \beta_1)$$

$$= \frac{\sin^2(\beta_1 - \beta_2)}{\sigma_1^2 r_1^2 r_2^2} + \frac{\sin^2(\beta_2 - \beta_3)}{\sigma_2^2 r_2^2 r_3^2} + \frac{\sin^2(\beta_3 - \beta_1)}{\sigma_3^2 r_3^2 r_1^2} \quad (24)$$

In the same procedure, one can compute one-step det(FIM) for four UAVs to find the corresponding sequence rule

$$\text{det} \mathbf{F} = \frac{\sin^2(\beta_1 - \beta_2) + \sin^2(\beta_1 - \beta_3) + \sin^2(\beta_1 - \beta_4)}{\sigma_1^2 r_1^2 r_2^2 r_3^2} + \frac{\sin^2(\beta_2 - \beta_3)}{\sigma_2^2 r_2^2 r_3^2} + \frac{\sin^2(\beta_3 - \beta_1)}{\sigma_3^2 r_3^2 r_1^2}$$

$$= \frac{\sin^2(\beta_1 - \beta_2)}{\sigma_1^2 r_1^2 r_2^2} + \frac{\sin^2(\beta_1 - \beta_3)}{\sigma_1^2 r_1^2 r_3^2} + \frac{\sin^2(\beta_1 - \beta_4)}{\sigma_1^2 r_1^2 r_4^2} + \frac{\sin^2(\beta_2 - \beta_3)}{\sigma_2^2 r_2^2 r_3^2} + \frac{\sin^2(\beta_3 - \beta_1)}{\sigma_3^2 r_3^2 r_1^2} + \frac{\sin^2(\beta_4 - \beta_1)}{\sigma_4^2 r_4^2 r_1^2} \quad (25)$$

When each term on the right-hand side of equations (24), (25) is defined as

$$\text{det} \mathbf{F}_{jk} = \frac{\sin^2(\beta_j - \beta_k)}{(\sigma_j \sigma_k r_j r_k)^2} \quad (26)$$

a sequence relationship of the one-step det(FIM) for multiple UAVs is produced as shown in Fig. 4.

Therefore, based on the sequence rule in Fig. 4, the general formulation of an analytical one-step det(FIM) can be induced as

$$\text{det} \mathbf{F} = \sum_{j=1}^{N-1} \sum_{k=j+1}^{N} \text{det} \mathbf{F}_{jk} \quad (27)$$
Cooperative guidance law

\[
\begin{array}{c|c|c|c|c}
N = 2 & N = 3 & N = 4 & \cdots & N = m \\
\det F & \text{sum} \ det F_1 & \text{sum} \ det F_1, \ det F_{12} & \cdots & \text{sum} \ det F_1, \ det F_{12}, \ldots, det F_{m-1}
\end{array}
\]

Fig. 4 Sequence relationship of the one-step-ahead det(FIM) for multiple UAVs

Now, it is possible to continue to derive the generalized cooperative guidance law using the aforementioned one-step det(FIM) relationship, equation (27). To construct the sequence relationship, the costates equation (17) can be modified as

\[
\begin{align*}
\lambda_{x1} &= \alpha_{12} \sin(2\beta_1 - \beta_2), \\
\lambda_{y1} &= -\alpha_{12} \cos(2\beta_1 - \beta_2) \\
\lambda_{x2} &= \alpha_{21} \sin(2\beta_2 - \beta_1), \\
\lambda_{y2} &= -\alpha_{21} \cos(2\beta_2 - \beta_1)
\end{align*}
\]

(28)

where

\[
\begin{align*}
\alpha_{12} &= -\frac{2 \sin(\beta_1 - \beta_2)}{\sigma_1^2 \sigma_2^2 r_1 r_2^2}, \\
\alpha_{21} &= -\frac{2 \sin(\beta_2 - \beta_1)}{\sigma_1^2 \sigma_2^2 r_1 r_2^2}
\end{align*}
\]

(29)

When multiple UAVs (more than two) are involved, since \( \alpha \) cannot be cancelled out unlike the previous two-UAV case, it should be retained. Next, for general formulation, the performance index must be modified using the generalized one-step det(FIM). Without loss of generality, as in the previous step, a sequence rule through expansion for several UAVs can be found using the modified costate equations. As a result, the final form of the generalized cooperative guidance law is expressed as

\[
u_j = \text{atan2}(-\lambda_{yj}, -\lambda_{xj})
\]

(30)

where

\[
\lambda_{xj} = \sum_{k=1, k\neq j}^{N} \alpha_{jk} \sin(2\beta_j - \beta_k)
\]

\[
\lambda_{yj} = \sum_{k=1, k\neq j}^{N} -\alpha_{jk} \cos(2\beta_j - \beta_k)
\]

(31)

and

\[
\alpha_{jk} = -\frac{2 \sin(\beta_j - \beta_k)}{\sigma_j^2 \sigma_k^2 r_j^2 r_k^2}
\]

(32)

Here, \( \alpha \) is a autonomous variable coefficient which plays the role of a autonomous variable coefficient reflecting the geometric relationship between different UAVs. It is worthwhile to note that even with many UAVs engaged in cooperative manoeuvres to estimate a target, all UAVs can be independently controlled by the proposed guidance command with the cooperative network. Accordingly, it is shown that the proposed guidance law is decentralized in its inherent structure. Although the previous study suggested the independent guidance law using a geometrical method [22], the guidance law should be reformulated to extend to other optimal manoeuvres which are subject to the varying number of UAVs. However, the new generalized cooperative guidance law proposed in this study does not require extra laborious effort for the increasing number of UAVs. Also, even if the number of UAVs changes by attack from enemy during any maneuver, the proposed guidance law can adaptively generate modified command for cooperative manoeuvres according to the number of UAVs in real-time. Therefore, the proposed formulation leads to an easy-to-implement feedback guidance command with negligible computational load. It should be noted that if multiple UAVs are on the same line to the target exactly, unexpected numerical singularity problem (\( \alpha = 0 \)) occurs, so the guidance law by equation (30) cannot generate correct command. However, this problem can be avoided easily by introducing a small perturbation to \( \Delta \beta \) at that instant.

4 RISK-ZONE AVOIDANCE PROBLEM

In order to apply the proposed guidance law for more practical scenarios, risk-zone avoidance problem should be considered. Obviously, even though a collision between UAVs could be considered, the collision may not be an issue in our problem because the guidance law generates the command to spread the difference angle between UAVs. Therefore, if the risk-zone is sensed in a certain range, the modified guidance law which avoids the invasion should be generated. To derive the modified guidance law, the previous performance index is redefined to account for the avoidance problem as follows

\[
f = -\xi \det \mathbf{F}(t + \Delta t) + (1 - \xi) \sum_{j=1}^{N_0} \sum_{k=1}^{N} f_{j\ell} \frac{\kappa}{\ell} + \int_{t}^{t+\Delta t} \lambda^T (f - \mathbf{X}) \mathrm{d}t
\]

(33)
where $N_O$ denotes the number of sensed risk zones in a certain range, $r_{ij0}$ the distance between $i$th sensed risk-zone, $j$th UAV, $\zeta \in [0,1]$ a weighting parameter for risk-zone avoidance, and $\kappa$ a constant for a repulsive force. Note that the parameter, $\kappa$ should be scaled according to the range term which appears in \( \text{det}(F_{IM}) \) expression. Hence, the modified guidance law can be derived by an optimization approach identical to the previous problem as follows

\[
\mathbf{u}_j = \text{atan}2(-\lambda_{yj}, -\lambda_{xj})
\]

(34)

where

\[
\lambda_{xj} = \zeta \left( \sum_{k=1}^{N} \alpha_{jk} \sin(2\beta_j - \beta_k) \right) - (1 - \zeta)\kappa \sum_{l=1}^{N_O} \frac{x_l - x_j}{r_{lj0}}
\]

\[
\lambda_{yj} = \zeta \left( \sum_{k=1}^{N} -\alpha_{jk} \cos(2\beta_j - \beta_k) \right) - (1 - \zeta)\kappa \sum_{l=1}^{N_O} \frac{y_l - y_j}{r_{lj0}}
\]

(35)

and

\[
\alpha_{jk} = -\frac{2 \sin(\beta_j - \beta_k)}{\sigma_x^2 \sigma_y^2 r_{jk}^2}, \ j, k \in [1, 2, \ldots, N], j \neq k
\]

Consequently, the modified guidance law is formulated as a weighted combination between the previous cooperative guidance law to maximize the information and a new command for avoidance. Also, the augmented command for the avoidance always generates heading input towards the reverse direction to the relative range vector between the UAV and the risk-zone.

5 SIMULATION RESULTS

To verify the cooperative guidance law in equation (30), numerical simulation studies are conducted. In the simulations, all UAVs are assumed to move at a constant speed of 10 m/s, and the target location is (0, 100) m. The characteristics of the bearing sensors of each UAV are supposed to be identical, and the measurement update rate is set to 10 Hz. Each UAV is considered to share states with each other and measured target information by cooperative network at every moment. Physical constraints on UAV maneuvers and the limitation of field-of-view are not considered in the simulation. In addition, if any UAV approaches near the target, the UAV is assumed to stay in the vicinity of the target by hovering or a coordinated turn with a small radius, but this scenario is not actually implemented in this study.

5.1 Verification of the cooperative guidance law

First, to demonstrate the performance of the cooperative guidance law, special initial conditions of multiple UAVs are regarded. When two UAVs are employed for this scenario, one is located at (0, 0) m and the other at (100, 100) m initially to meet the orthogonal condition for optimal maneuver. Similarly, in the three-UAV case, the first UAV is situated at (0, 0) m, the second at (70.7, 150) m, and the final UAV is at (−70.7, 150) m symmetrically to cope with the optimal condition of three UAVs from the start. Figures 5(a) and (b) show the resultant trajectories in these special cases according to the optimal conditions satisfied continuously for the orthogonal or 2n/3 maneuver. These results are consistent with the solutions of path planning by the direct optimization method, considering the total information (not one-step information), based on the established sequential quadratic programming approach [25]. If each UAV cannot share the information of the target and its own states, the observability of each
trajectory is severely degraded. In this case, however, since each UAV can share the information, the results can maximize the target information as an optimal solution. From these results, one can see that the new cooperative guidance law indeed reconstructs the characteristics of optimal manoeuvres.

Now that the cooperative guidance law has been demonstrated, a general situation with non-optimal initial conditions needs to be considered. To analyse this case, each initial position of the UAVs is assigned as \((-10, 0)\) m and \((10, 0)\) m for the two-UAV case. As shown in Fig. 6, both UAVs approach the target in a symmetric formation in Fig. 6(a), and the difference between the bearing angles gradually approaches \(\pi/2\). As the initial difference between the bearing angles is very small, the command for orthogonality is primarily issued, and both UAVs are spread out widely at the initial stage. Because the information in range direction is relatively poor over the cross-range direction due to the characteristics of a bearing sensor, this initial spread behaviour should be expected to increase the information in range direction. On the other hand, when the difference between the bearing angles is close to orthogonal, the forward manoeuvre command is generated dominantly because the range direction of error covariance becomes sufficiently small as shown in Fig. 2. Even if the initial conditions are not symmetric, the orthogonal manoeuvres are generated also. Additionally, even if the target manoeuvres with an arbitrary motion, because the proposed guidance law attempts to generate commands to maximize the information of the next step.

**Fig. 6** General case (initial positions of two UAVs not satisfying the optimal condition) with a fixed target

**Fig. 7** Results of the case when one among the three UAVs is dissipated
every time, it can produce an explicit trajectory with the similar trend as optimal maneuver (but, these results are not presented here).

Also, as mentioned previously, the cooperative guidance law can generate commands adaptively according to the number of UAVs in real-time without reformulation of the guidance law. In order to verify this, one of three UAVs is assumed to be attacked and reformulation of the guidance law. In order to verify according to the number of UAVs in real-time without guidance law can generate commands adaptively results are not presented here).

Also, as mentioned previously, the cooperative

The performance index in the RHC formulation is defined as

\[ J = -\det F(t_{g+h}) + \int_{t_g}^{t_{g+h}} \lambda^T (f - X) \, \mathrm{d}t \]  

\[ (36) \]

Though the trend of trajectories in the case of RHC based upon the accumulated FIM is analogous to the proposed guidance law, the amount of spread becomes different according to the time horizon. In the short horizon case (0.1–5 s), since the wider spread maneuvers are generated from the initial stage, total information get degraded due to longer final range to the target consequently. However, in the longer horizon case, because the obtained information by range reduction is larger than by increase in the difference between each bearing angle, the amount of spread becomes relatively smaller.

This analysis can be explained mathematically as follows. Accumulated FIM can be expressed as

\[
F(t_{g+h}) = \begin{bmatrix}
F_{11} + I_{11} & F_{12} + I_{12} \\
F_{12} & F_{22} + I_{22}
\end{bmatrix}
\]  

\[ (37) \]

where \( F_{mn} \) is the accumulated element of FIM, and \( I_{mn} \) represents element of the pseudo information matrix during the horizon time.

5.2 Comparison of the cooperative guidance law with RHC

Although the proposed approach in this article is based on one-step det(FIM), the results which are not optimal seems to show the similar trend to the previous studies. Therefore, to compare with the previous approach which does not consider the one-step but accumulated total information, the results by the RHC are described in Fig. 8. The number written after ‘RHC’ in Fig. 8 indicates various horizon times (s).

\[
F_{(t_g+h)} = \begin{bmatrix}
\frac{\sin^2 \beta_1(t_1)}{\sigma_1^2(t_1)\sigma_2^2(t_1)} & \frac{\sin^2 \beta_2(t_1)}{\sigma_2^2(t_1)\sigma_1^2(t_1)} & -\frac{\sin \beta_1(t_1) \cos \beta_1(t_1)}{\sigma_1^2(t_1)\sigma_2^2(t_1)} & -\frac{\sin \beta_2(t_1) \cos \beta_2(t_1)}{\sigma_2^2(t_1)\sigma_1^2(t_1)} \\
-\frac{\sin \beta_1(t_1) \cos \beta_1(t_1)}{\sigma_1^2(t_1)\sigma_2^2(t_1)} & -\frac{\sin \beta_2(t_1) \cos \beta_2(t_1)}{\sigma_2^2(t_1)\sigma_1^2(t_1)} & \frac{\cos^2 \beta_1(t_1)}{\sigma_1^2(t_1)\sigma_2^2(t_1)} & \frac{\cos^2 \beta_2(t_1)}{\sigma_2^2(t_1)\sigma_1^2(t_1)}
\end{bmatrix}
\]  

\[ (38) \]
To analyse the direction of generated command, the costates should be derived

\[ \lambda_{X(t_{g+h})} = \frac{\partial}{\partial X(t_{g+h})} \det F(t_{g+h}) \]
\[ = (F_{110} I_{2y} + I_{11} F_{220} - 2F_{120} I_{12}) \]
\[ + (I_{11} I_{22} - I_{12}^2) \]  
(39)

where prime sign denotes partial derivatives with respect to the states. Note that \( F_{120} \) is close to zero. Hence

\[ \lambda_{X_i(t_{g+h})} = \frac{\partial}{\partial x_i} \det F(t_{g+h}) = A_0 + A \]
\[ \lambda_{X_j(t_{g+h})} = \frac{\partial}{\partial y_j} \det F(t_{g+h}) = B_0 + B \]  
(40)

where

\[ A_0 = \frac{1}{f_j} (-4 \cos \beta_j \sin^2 \beta_j (F_{110} - F_{220}) + 2 \cos \beta_j F_{110}) \bigg|_{t=t_{g+h}} \]
\[ B_0 = \frac{1}{f_j} (4 \cos^2 \beta_j \sin \beta_j (F_{110} - F_{220}) + 2 \sin \beta_j F_{220}) \bigg|_{t=t_{g+h}} \]
\[ A = -\frac{2 \sin(\beta_j - \beta_k)}{\sigma_f^2 \sigma_k^2} \sin(2\beta_j - \beta_k) \bigg|_{t=t_{g+h}} \]
\[ B = \frac{2 \sin(\beta_j - \beta_k)}{\sigma_f^2 \sigma_k^2} \cos(2\beta_j - \beta_k) \bigg|_{t=t_{g+h}} \]  
(41)

To analyse the above equations easily, only UAV#1 at the initial stage, \((\beta_1 > \pi/4, r_1 > 0)\), is handled considering a small horizon time. Note that \( F_{110} > F_{220} \). Also, \( A \) and \( B \) are identical to the costates of the proposed guidance law from this conditions. Owing to \( B_0 > 0, B < 0 \), therefore, \( \lambda_{X_i} \) in this case is reduced relatively rather than the one of the guidance law \((i.e. \Delta \lambda_y < 0)\). On the other hand, \( A_0 \) equation can be modified as

\[ A_0 = \frac{1}{f_j} (2 \cos \beta_j F_{110}(-2 \sin^2 \beta_j + 1) \]
\[ + 4 \cos \beta_j \sin^2 \beta_j F_{220} \]  
(42)

Because of \(-1 \leq (-2 \sin^2 \beta_j + 1) < 0\), \( \lambda_{X_k} \) is reduced or increased \((\Delta \lambda_x < 0 \text{ or } \Delta \lambda_x > 0)\). Since even if the \( \lambda_{X_k} \) is reduced, it follows that \( \Delta \lambda_j > \Delta \lambda_x \). Therefore, the results of the accumulated information case create more spread at the initial stage.

Based on such a resultant trajectory, FIM seems to exhibit an analogous trend too. In the longer time horizon case, improved final FIM can be obtained as the final relative distance to target gets shorter. In the initial stage, however, since the difference between each bearing angles becomes smaller, the initial information of the longer time horizon case deteriorates relatively. Hence, in the case of the estimation process, because initial convergence rate gets slower, it cannot be concluded that the longer horizon leads to better results, considering only the final FIM. Furthermore, the longer horizon time, the more computation load is required. Therefore, based on these characteristics, since the proposed guidance law produces good performance similar to the RHC approach without computation load, it could be a practically effective method in real-time operation.

5.3 Sensitivity analysis to initial estimation errors

The proposed guidance law is derived based on the FIM considering true target states, but the true target states are unknown due to the measurement noise in practical implementation. Therefore, estimated target states should be used instead of the true target states in order to generate the command. This estimation error makes the difference from the ideal command based on the true target states. If the sensitivity according to the estimation error is not small, the proposed approach is not a proper method for actual operation. Hence, analysis on the sensitivity about the estimation error is needed.

To estimate the target states, Extended Kalman Filter algorithm is utilized because the measurement equation is non-linear. Negligible process noise is assumed (due to the stationary target), hence \( Q \) is fixed as \( 0.0001 \times I_2 \text{ (m/s)}^2 \), and the measurement noise is set to \( R = 0.052 \times I_2 \text{ rad}^2 \). The results according to the various initial estimation errors are described as shown in Fig. 9. The number behind ‘Err’ in Fig. 9 represents the percentage of initial error including both x- and y-axes. The bold solid line represents the results by true target states. Owing to the characteristics of a bearing sensor, when the multiple sensors are applied, the cross-range estimation converges rapidly without high sensitivity about the estimation error. On the other hand, because the range estimation is worse, the sensitivity of the range direction becomes relatively higher. Therefore, as shown in Fig. 9(a), even if the initial estimation errors in the x-axis are biased, the near-unbiased trajectories of both UAVs are obtained in all cases due to rapid convergence in x-axis, but the amount of spread becomes slightly different due to degraded estimation performance in range direction. Hence, the difference in the amount of spread makes the FIM gap as shown in Fig. 9(b). Nevertheless, the resultant trajectory and FIM show a similar trend with small difference; so, one can conclude that the sensitivity using the estimated target states may not be significant from practical perspective.
5.4 Risk-zone avoidance problem

In order to prove that the modified guidance law can avoid any risk-zone, three UAVs are adopted, and the target is assumed to be stationary. Risk zones are located at arbitrary positions and these areas are supposed to be sensed in a certain range. In this simulation, it is considered that the UAVs can perceive the areas if the relative distance between the UAV and the area is equal or less than 10 m, that is \( r_{ij} = 10 \). Also, \( \zeta \) is tuned not to approach the risk-zone within a 5 m radius, and \( \zeta \) is selected as 0.5 to weight equally the guidance law for maximizing the information and the augmented command for avoidance. Figure 10(a) shows the original trajectories without avoidance (thin line) and the modified trajectories (bold line), whereas Fig. 10(b) presents the input histories, difference between the bearing angles, and the minimum distance from the risk zones. As shown in Fig. 10(a), UAVs perceive the zones in 10 m radius area (bright circles), and then manoeuvre to avoid the invasion into the risk-zone (dark circles) by passing. The grey
squared region representing ‘minimum distance from the risk zones’ in Fig. 10(b) describes the invasion-possible range. It is shown that after escaping from the invasion-possible area (bright circles), only pure cooperative guidance law is generated. Also, as one can see, if the trajectory of one UAV is changed due to the avoidance maneuver, other trajectories are affected and then modified as well. Since only UAV#1 maneuvers to avoid some invasions initially, even if other UAVs have no possibility of any invasion, the trajectory of UAV#3 is changed slightly for the geometric relationship. Therefore, it is confirmed that the modified guidance law serves both purposes to maximize the information of the target and to avoid the risk-zone.

6 CONCLUSIONS

The new guidance law for cooperative estimation by multiple UAVs using only bearing sensors with cooperative network was demonstrated. The analytical one-step determinant of the FIM associated with the trend of optimal maneuver at any instant was applied to an optimal target estimation problem, and then the cooperative guidance law for two UAVs in a feedback form was constructed using an indirect optimization method. To prove the validity of the proposed guidance law, the results were compared with the previous approach which considers accumulated information not one-step, using RHC approach. Also, the guidance law turned out to be a weighted combination of the orthogonal and forward flights, which is consistent with the trend of optimal maneuvers. Because such an observation helps us to anticipate how command is generated, it is useful and important for practical guidance laws. For applications to more number of UAVs, a general formulation of the cooperative guidance law for N UAVs was developed, which seems to be a weighted summation of each guidance law inter-connected with geometric relationship. Hence, since the general formulation was not changed by the number of UAVs, the proposed guidance law could be employed without reformulation even if the number of UAVs was varied during maneuvers. In addition, modified guidance law for restricted region avoidance problem was proposed in the form of a weighted combination between commands for information and avoidance, and verified by simulation.

Consequently, due to its simple feedback structure and capability of handling several UAVs without incurring extra computational load, the proposed approach can be regarded as a viable and effective solution in the construction of explicit trajectories in real-time estimation or tracking operations.

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APPENDIX

Notation

- $f$: dynamic equations of UAVs
- $F_j$: FIM of $j$th UAV
- $F_{\text{acc}}$: accumulated element of FIM
- $H$: Hamiltonian
- $I$: pseudo information matrix during the horizon time
- $I_{mn}$: element of the pseudo information matrix
- $J$: performance index
- $N$: the number of UAVs
- $N_D$: the number of sensed risk zones in a certain range
- $P$: error covariance matrix
- $Q$: process noise
- $r_j$: relative distance between $j$th UAV and a target
- $r_{jh}$: distance between $h$th sensed risk zone and $j$th UAV
- $R$: measurement noise
- $u_j$: heading command of $j$th UAV
- $v$: velocity of UAVs
- $x_j, y_j$: 2D $x$-position and $y$-position of $j$th UAV
- $x_T, y_T$: 2D $x$-position and $y$-position of a target
- $X$: state vector of UAVs
- $\alpha$: variable coefficient of the cooperative guidance law
- $\beta$: set of bearing measurements
- $\beta_j$: bearing measurement of $j$th sensor
- $\zeta$: weighting parameter for risk-zone avoidance problem
- $\kappa$: constant for a repulsive force for risk-zone avoidance problem
- $\lambda$: lagrange multiplier
- $\nu$: measurement noise of bearing sensor
- $\sigma_j^2$: noise covariance of $j$th sensor
- $\phi$: soft constraint in optimal problem
- $\Phi$: state transition matrix
- $\chi_T$: state vector of a target